This is math? Finding a way into mathematical thinking via puzzles & games (K-6)

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I've found that mathematical/logical puzzles and games appeal to a very wide range of people of all ages. In the classroom, they draw in students who are not normally interested in mathematics, they give students who may not excel at algorithmic manipulation a chance to show how well they can think logically to solve difficult problems, and they provide interesting challenges for students who find the curriculum straightforward.

While on sabbatical last year, I had the great joy of sharing mathematical/logical puzzles and games with over 4100 K-12 students and hundreds of teachers, either in the classroom or in workshops.

Elementary teachers who invited me into their classrooms were pleased and often surprised at how much excitement, higher-level thinking, and tenacity their students displayed. They liked the fact that the puzzles are graduated in difficulty, allowing students to work at their own pace while engaged in a common activity, and often incorporate brightly-coloured manipulatives, none of which are expensive or difficult to make. They loved it that some of their students figured out a puzzle before they themselves did. Many teachers told me something along the lines of "I knew that Kim would enjoy these games, but I had no idea that Sam would be so good at them", or "That student hasn't asked one question all year; she actually asked you something!" A grade 4 student surprised me by articulating a clear approach that allowed him to solve a 4x4 Latin square in very little time. One child with cerebral palsy, who struggles with letters and numbers alike, amazed his teacher by being among the fastest in the class to create a 4x4 Euler square.

At the end of one of my classroom visits, an astute young grade 1 teacher had her children take it in turn to tell us what they had learned about doing math from our activities. I was delighted that several of them said variations of "start with what you know for sure", "be sure you know what the rules are", and "don't be afraid to clear off the whole puzzle and start again". I wish I had the complete list, as it provided a wonderful summary of general problem-solving approaches.

When working with people of any age I have found it most effective to first display a completed puzzle and ask the group to figure out what the rules must be. Then we do at least one puzzle together; I insist that elementary students make their suggestions using words rather than gestures, for example "10 goes above 8", "7 goes in the third box from the right in the bottom row", "3 fits diagonally between 2 and 4", etc. Thus we get some inductive reasoning, the use of common language to describe spatial relationships, and the necessity of precise description. It usually takes only one or two examples of unclear descriptions for students to start to be very precise, to the delight of their teacher.

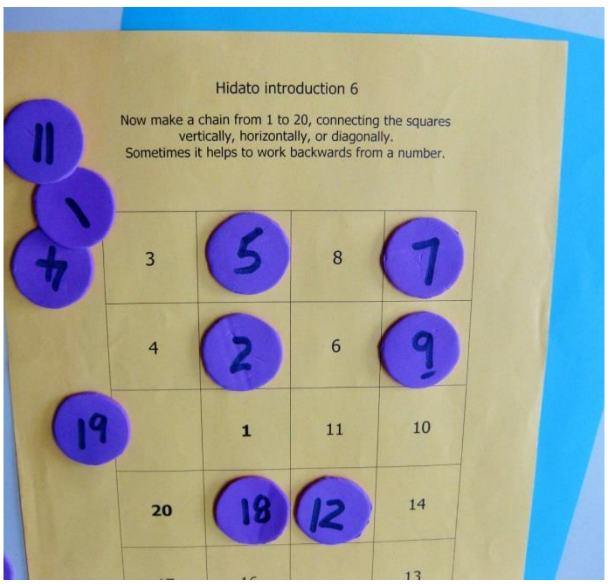
Here are some of the puzzles that were highly successful with elementary school students – for more, come to the workshop for secondary students or check out my Dropbox.

Hidato

This is a lovely puzzle for young children and an excellent warm-up for older ones. Even kindergarten children pick up on it quickly, as in order to get started it is necessary only to know the numbers 1-10 in order. To introduce the puzzle to primary grades I use brightly-coloured counters and small puzzles. It doesn't usually take the grade 2s and 3s long to graduate to the bigger pencil-and-paper versions. Once they figured out the first few examples, students in intermediate grades seemed to solve the puzzles almost as quickly as I could hand them out, demanding ever harder ones.

	19	16	14	
	17		11	13
21	**1**	10		
25			7	5
23		3		

Make a chain from 1 to 25, connecting the squares vertically, horizontally, or diagonally.



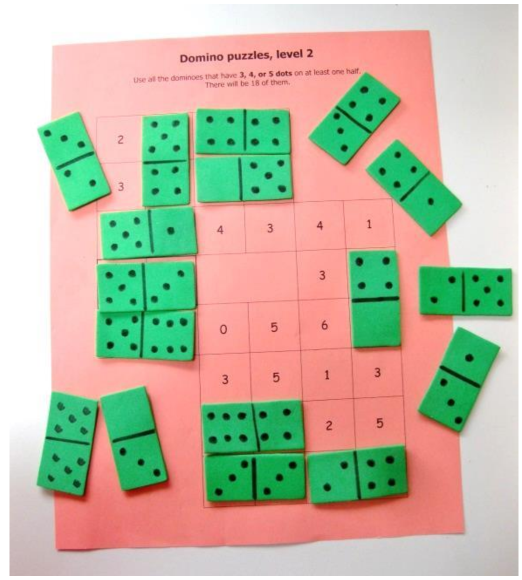
I was very impressed that within 15 minutes many elementary students of all ages could produce fairly long chains of reasoning along the lines of, "I can't put the 12 here, because if I did, then the 13 would have to go here, which would force the 16 over there, so the 18 goes there, and then this square in the corner would have no one to connect to". They took great delight in informing me of supposed misprints in my puzzles, only to track down their own errors as they explained the problem to me. As I often stress to my own classes, the ability to find our own errors is a big part of doing mathematics.

Domino puzzles

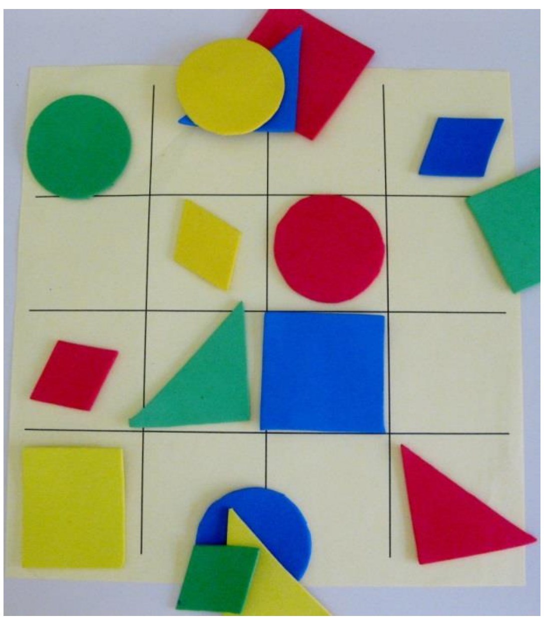
All the examples of domino puzzles I have seen involve at least all 28 dominoes of a standard set. These can be very hard, are off-putting for many people, and certainly wouldn't appeal to most elementary students. I reduced the number of dominoes to 13 for the lowest level, appropriate for grades K-4, and 18 for the next level, appropriate for grades 3 and up. Within a level I've tried to graduate the level of difficulty, although that can be hard to gauge.

The dominoes are made from coloured plastic foam. Children like having their own colour to play with, and having different colours makes it less likely that the sets get mixed up with each other.

In my experience, the biggest hurdle in solving these puzzles is getting past the stage of trying to place the first domino that comes to hand, rather than looking over the puzzle for the most obvious places to start, then finding the dominoes to go there. Some children seem to do this naturally (or perhaps have experience with jigsaw puzzles), while some need nudging. Analysing a problem before trying to tackle it is something we all want our students to do.



Latin & Euler squares



This set of puzzles has many things to recommend it, among them: it is visually appealing, with bright colours and distinct shapes rather than numbers or letters; it requires puzzlers to be very clear about which rules they are using; there are thousands of solutions for each level; and solutions can be found via many approaches. Also, many other puzzles become accessible once students have internalised the idea of a Latin square. Examples include Kenken, Towers, Neighbours, and Kakuro.

Unlike the way I introduce other puzzles, I usually put these rules – one level at a time – on the board, as the first students complete each level. That way each student can double-check the rules that s/he is currently using. I think it is useful for students to see the progression in complexity. If I don't put up the next rule quickly enough, students will often suggest it.

Getting the idea across for level 1 (below) often requires several re-statements, as some people oversimplify and others make it more complicated than necessary. I have discovered that, in any age group from 5 – 80, there are people who can hear a rule and proceed to use it, others who can read it and proceed to use it, others who read a rule and need to re-state it in their own words before using it, and some who benefit from hearing the re-statement from their peers.

Latin squares: (Sudoko players will find the idea familiar)
You'll need *n* different colours of pieces and *n* of each colour: to start with, try 3 different colours with 3 of each colour. Then try 4 different colours, with 4 of each.

level 0 (K/1): Arrange the pieces so that each row is all one colour, or each column is.
level 1 : Arrange the pieces so that each colour turns up once only in each row and in each column. Restatement: *We want four colours in each row and in each column.*
level 2 : Arrange the pieces so that each colour turns up once only in each row, each column, and on the two main diagonals. (not always possible; it is interesting to try to figure out which sizes will allow for this)
level 3: As for level 2, but also once only on all diagonals! (also not always possible)

Euler squares:
You'll need four shapes in four colours each (16 pieces) plus a 4x4 grid.

level 0 : name the shapes, figure out which are the rows and which are the columns on the grid
level 1 : make every row a different colour and every column a different shape
level 2 : no **colour** appears twice in a row or in a column (*4 colours in each row, each column)*
level 3 : no colour appears twice in a row, a column or in either of the two main diagonals
level 4 : no **shape** appears twice in a row, column or main diagonals (many of us find that this is harder to do than the same challenge using colours)
level 5 : no **colour or shape** appears twice in a row or column
level 6 : no colour or shape appears twice in a row, column or main diagonal

For students who complete level 6 well ahead of the rest of the class, I make their own personal puzzle by giving them a few pieces as a starting place. It would also be possible to give a particular configuration as a starting point and ask the class to all solve the same puzzle.

When I work with K-1 students I usually start with 3x3 Latin squares; once they sort out the rules, they are keen to try the 4x4 Latin squares. Over a couple of sessions I think it would be possible for them to create 4x4 Euler squares and astonish their parents. (Many adults find these puzzles difficult.) I've seen grade 2 students in a particularly focussed but otherwise ordinary class progress in less than 25 minutes from Latin squares to 4x4 Euler squares. They insisted on my staying well past the allotted time so that they could solve these puzzles.

There are many more puzzles that can be modified for young students. I think the most important thing is that a puzzle be interesting in its own right and not obviously educational!

Many of my puzzles are modified from on-line sites. Brainbashers and Simon Tatham's Portable Puzzle Collection are two excellent sources. Some are inspired by puzzles from assorted books, such as Moscovich's *1000 Playthinks of Art, Mathematics & Science* (Workman Publishing, 2001), and those spiral-bound puzzle books one finds in Coles/Indigo bookstores. A few are based on games one can buy commercially, such as SET.

Send me an e-mail (susan.milner@ufv.ca) if you would like access to my Dropbox , which contains these and many more puzzles and games at different levels which you can download and copy for your classes or use as templates to make more of your own. It also contains annotated lists of on-line resources and commercial games which I have tested on people of all ages.